ON THE MOTION OF A BODY OF REVOLUTION BOUNDED BY A SPHERE, ON A SPHERICAL BASE

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PMM Vol. 30, No. 5, 1966, pp. 934-935

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(Received November 10, 1965)

This paper is closely related to [1 to 4]. The problem considered here is that of rolling of the body of revolution bounded by a sphere, on another fixed sphere, in case when the force function of the given forces is a function of u coordinate only. In other words, we shall investigate the equations*

$$\frac{d}{du}\frac{\partial\Theta}{\partial\sigma}+\frac{R}{R_c}\frac{\partial\Theta}{\partial n}-n\,[M(R^2+l^2+2Rl\cos u)-A]\,\frac{R-R_c}{R_c}+\sigma\,[M(R^2+l^2+2Rl\cos u)+$$
$$+A]\operatorname{ctg}u+MlR\sin u\sigma+\varkappa\frac{R-R_c}{R_c}\cos u=0 \qquad (1)$$
$$\frac{d}{du}\frac{\partial\Theta}{\partial n}+\frac{R}{R_c}\sigma\,[M(R^2+l^2+2Rl\cos u)+A]-\frac{R}{R_c}\frac{\partial\Theta}{\partial\sigma}+MlR\sin un+\varkappa\frac{R-R_c}{R_c}\sin u=0$$

integration of which is equivalent to the problem of determining the magnitudes u, v, σ, π and n in the above problem.

Before anything else, we shall write the above equations in the more detailed form, assuming that the unknown functions are n and $r=-\sigma\sin u+n\cos u$ (projection of the angular velocity of the body on the Oz-axis), while $\gamma=\cos u$ is the independent variable. This yields the equalities

$$[M(R^2+l^2\gamma^2+2Rl\gamma)+A+(C-A)(1-\gamma^2)]\frac{dr}{d\gamma}+[(c-A-Ml^2)\gamma-MlR]\frac{R-R_c}{R_c}r-$$
$$-[MRl\gamma^2+(A+Ml^2+MR^2)\gamma+MlR]\frac{dn}{d\gamma}-M(R+l\gamma)\frac{R^2}{R_c}n+\varkappa\frac{R-R_c}{R_c}\gamma=0 \qquad (2)$$
$$[MlR-(C-A-Ml^2)\gamma]\frac{dr}{d\gamma}+(C-A-Ml^2)\frac{R-R_c}{R_c}r-(A+Ml^2+MlR\gamma)\frac{dn}{d\gamma}-$$
$$-Ml\frac{R^2}{R_c}n+\varkappa\frac{R-R_c}{R_c}=0$$

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* First and third equation of (2.5) in [3], where divided by $u^{\cdot}$.

Multiplying the second equation by $-\cos u$ and adding it to the first one, we obtain

$$[C+MR(R+l\gamma)]\frac{dr}{d\gamma}-MR(l+R\gamma)\frac{dn}{d\gamma}-MlR\left(\frac{R}{R_c}-1\right)r-M\frac{R^3}{R_c}n=0 \qquad (3)$$

while from the second equation of (2) together with (3), we obtain

$$\frac{dn}{d\gamma}=\frac{1}{AR}[R(A-C)\gamma-lC]\frac{dr}{d\gamma}+\frac{C-A}{A}\left(\frac{R}{R_c}-1\right)r+\frac{\varkappa}{A}\left(\frac{R}{R_c}-1\right)$$

$$\frac{d^2n}{d\gamma^2}=\frac{1}{AR}[R(A-C)\gamma-lC]\frac{d^2r}{d\gamma^2}+\frac{C-A}{A}\left(\frac{R}{R_c}-2\right)\frac{dr}{d\gamma} \qquad (4)$$

Differentiating (3) with respect to γ and inserting (4), we find the following non-homogeneous linear second order equation for r

$$[AC+MR^2A(1-\gamma^2)+MC(l^2+2lR\gamma+R^2\gamma^2)]\frac{d^2r}{d\gamma^2}+3[CMR(l+R\gamma)-AMR^2\gamma]\frac{dr}{d\gamma}-$$

$$-MR^2(C-A)\left(\frac{R^2}{R_c{}^2}-1\right)r-MR^2\varkappa\left(\frac{R^2}{R_c{}^2}-1\right)=0 \qquad (5)$$

Corresponding homogeneous equation can be written in the form (a_1, a_2, a_3, b_1, b_2 and c_1 are constants)

$$(a_1x^2+a_2x+a_3)\frac{d^2r}{dx^2}+(b_1x+b_2)\frac{dr}{dx}+c_1r=0 \qquad (6)$$

and is of Fuchsian type [5]. Its singularities are $c'=\infty$ and the roots a' and b' of the equation

$$a_1x^2+a_2x+a_3=0 \qquad (7)$$

Assuming that $r=z(y)$ and $x=a'+(b'-a')y$ (in case $a'\neq b'$), we obtain a hypergeometric equation

$$y(1-y)\frac{d^2z}{dy^2}+\left(\frac{3}{2}-3y\right)\frac{dz}{dy}-\left(1-\frac{R^2}{R_c{}^2}\right)z=0 \qquad (8)$$

Now, first equation of (4) together with the relation $r=-\sigma\sin u+n\cos u$, enables us to find n and σ in terms of u. These can then be inserted into the force integral. This reduces the problem of determination of u as a function of time, to a quadrature, and consequently settles the problem of determination of u, v, σ, τ and n.

In this manner the problem of rolling of a body of revolution bounded by a sphere on another fixed sphere when the given force function depends only on u, is reduced to integration of one Riccati equation and to quadratures [2 and 3].

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Translated by L.K.